Actual Cause and Chancy Causation in *stit*

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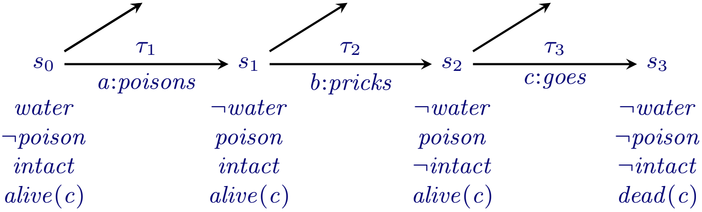
An example of 'proximate cause'



$s_0 \xrightarrow[a{:}poisons]{\tau_1} s_1 \xrightarrow[b{:}pricks]{\tau_2} s_2 \xrightarrow[c{:}goes]{\tau_3} s_3$

$water$	$\neg water$	$\neg water$	$\neg water$
$\neg poison$	$poison$	$poison$	$\neg poison$
$intact$	$intact$	$\neg intact$	$\neg intact$
$alive(c)$	$alive(c)$	$alive(c)$	$dead(c)$

J.A. McLaughlin. Proximate Cause. *Harvard Law Review* 39(2):149–199 (Dec. 1925)

Dretske: *structuring* vs *triggering* causes

Logics of action/agency

- ‘Seeing to it that’ (Belnap, Perloff, Horty, . . . , Broersen, . . .)

  $$[stit\, x]\varphi \qquad [dstit\, x]\varphi$$

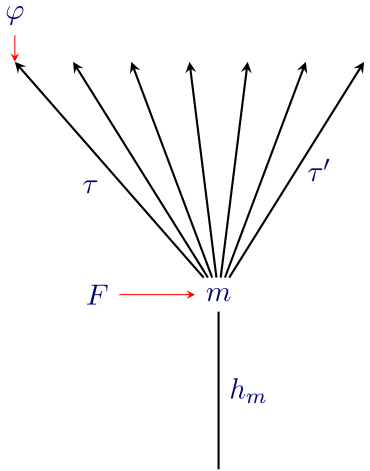
  Many versions!

- Ingmar Pörn’s logic of ‘brings it about’

  $$\mathrm{E}_x\, F$$

A common informal reading

$$[stit\, x]\varphi \quad — \quad x \text{ causes/is responsible for } \varphi$$

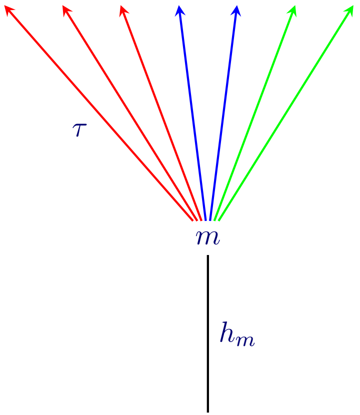


moment-histories
branches
continuations
transitions
trajectories
traces

$\tau \sim \tau'$

$alt(\tau)$ $[\tau]^{\sim}$

$\|\varphi\|$ — a set of moment-histories/transitions/trajectories

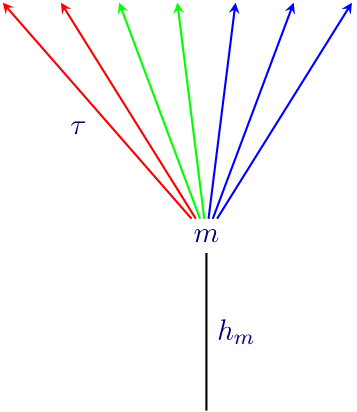


alt^{m}_{x} — partition

$\sim_x$ — equivalence relation

$alt_x(\tau)$ $[\tau]^{\sim_x}$

$alt_x(\tau)$ — the action performed by x in τ



alt_y^m — partition

$\sim_y$ — equivalence relation

$alt_y(\tau)$ $[\tau]^{\sim_y}$

$alt_y(\tau)$ — the action performed by y in τ

Generalisation

For every non-empty subset $G \subseteq Ag$:

$$\sim_G \;=_{\text{def}}\; \bigcap_{x \in G} \sim_x$$

$$alt_G(\tau) = \bigcap_{x \in G} alt_x(\tau)$$

Generalisation

For every non-empty subset $G \subseteq Ag$:

$$\sim_G \; =_{\text{def}} \; \bigcap_{x \in G} \sim_x$$

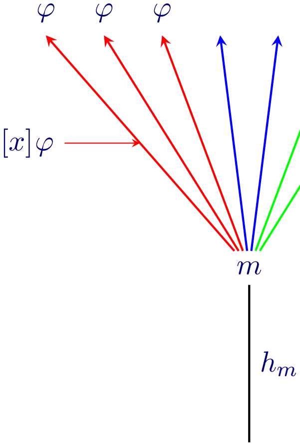
$$alt_G(\tau) = \bigcap_{x \in G} alt_x(\tau)$$

Axiomatisation:

$\Box$ type S5		
$[G]$ type S5		
$\Box\varphi \to [G]\varphi$		('necessity')
$[G]\varphi \to [H]\varphi$	$(G \subseteq H)$	('superadditivity')
$[\emptyset]\varphi \leftrightarrow \Box\varphi$		

Exactly the same as 'distributed knowledge'

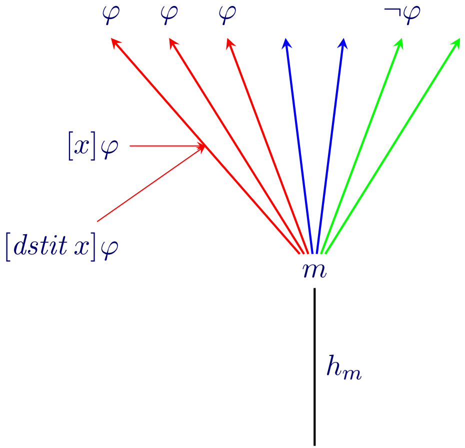
$\Diamond$ and $\langle G \rangle$ are the duals



alt_x^m — partition

$\sim_x$ — equivalence relat

$alt_x(\tau)$ $[\tau]^{\sim_x}$



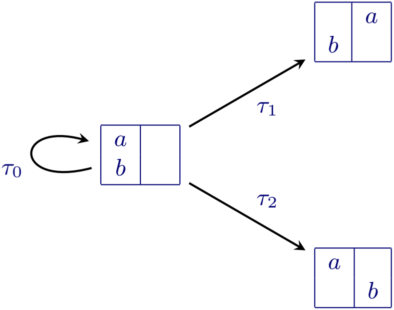
alt_x^m — partition

$\sim_x$ — equivalence relat

$alt_x(\tau)$ $[\tau]^{\sim_x}$

$$[dstit\,x]\varphi \;=_{\mathrm{def}}\; [x]\varphi \wedge \neg\Box\varphi$$

Example



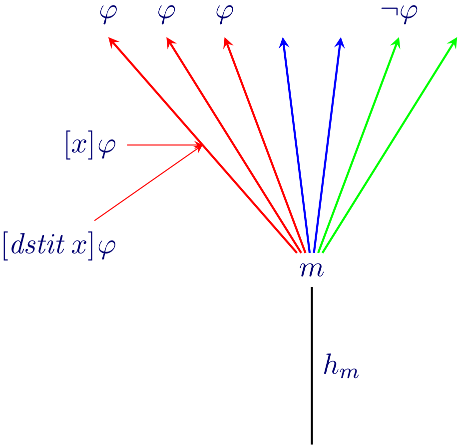
$\tau_0 \sim_b \tau_1$

$\tau_0 \sim_a \tau_2$

Possible actions by a: $\{\,\{\tau_0, \tau_2\},\ \{\tau_1\}\,\}$

Possible actions by b: $\{\,\{\tau_0, \tau_1\},\ \{\tau_2\}\,\}$

The (deliberative) stit



$Choice_x^m$

$\cancel{alt_x^m}$ — partition

$\sim_x$ — equivalence relation

$\cancel{alt_x(\tau)}$ $[\tau]^{\sim_x}$

$Choice_x^m(\tau)$

And add *'agent independence'* — a constraint on $Choice_x^m$.

stit-independence

> *'. . . simultaneous actions by distinct agents must be independent in the sense that the choices of one agent cannot affect the choices available to another; at each moment, each agent must be able to perform any of his available actions, no matter which actions are performed at that moment by the other agents.' (Horty, 2001)*

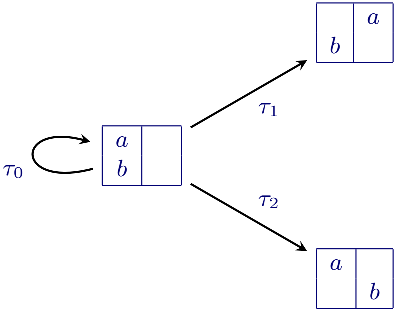
For all τ and all pairs of distinct agents $x \neq y$:

$$Choice_x^m(\tau) \cap Choice_y^m(\tau) \neq \emptyset$$

For all τ and all distinct agents $x, y, \ldots z$:

$$Choice_x^m(\tau) \cap Choice_y^m(\tau) \cap \cdots \cap Choice_z^m(\tau) \neq \emptyset$$

Example (rooms)



$\tau_0 \sim_b \tau_1$

$\tau_0 \sim_a \tau_2$

Possible actions by a: $\{\,\{\tau_0, \tau_2\},\ \{\tau_1\}\,\}$

Possible actions by b: $\{\,\{\tau_0, \tau_1\},\ \{\tau_2\}\,\}$

No *stit*-independence: $\{\tau_1\} \cap \{\tau_2\} = \emptyset$

Possible actions by $\{a, b\}$: $\{\,\{\tau_0\},\ \{\tau_1\},\ \{\tau_2\}\,\}$

stit-independence

$$\models\ (\Diamond[x_1]\varphi_1 \wedge \ldots \wedge \Diamond[x_n]\varphi_n) \to \Diamond\big([x_1]\varphi_1 \wedge \ldots \wedge [x_n]\varphi_n\big)$$

$$\models\ [x][y]\varphi \to \Box\varphi \qquad \text{for all } x \neq y$$

$$\models\ \neg[\mathit{dstit}\, x][\mathit{dstit}\, y]\varphi \qquad \text{for all } x \neq y$$

More generally

$$\models\ [G \cap H]\varphi \leftrightarrow [G][H]\varphi$$

stit-independence: Rationale

A technical requirement, easily avoided, but also:

agency implies free choice '. . . a possible choice for an agent at a moment should be considered as a *real* alternative for the agent, i.e., the realization of that alternative is exclusively up to the agent.' (Xu)

simultaneous choices are independent '. . . the only way that the choices open to one agent can depend on the choices open to another agent is if the one agent's choices lie in the causal past of those of another agent.' (Belnap and Perloff)

Abstract semantics

$$\mathcal{M} = \langle W, \sim, \{\sim_x\}_{x \in Ag}, V \rangle$$

$\sim_x \ \subseteq \ \sim$ equivalence relations on W

$\mathcal{M}, \tau \models \Box\varphi$ iff $\mathcal{M}, \tau' \models \varphi$ for every τ' such that $\tau \sim \tau'$

$\mathcal{M}, \tau \models [x]\varphi$ iff $\mathcal{M}, \tau' \models \varphi$ for every τ' such that $\tau \sim_x \tau'$

Abstract semantics

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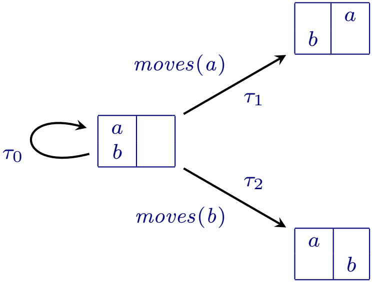
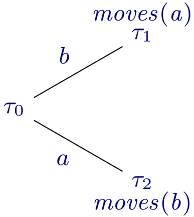




Example (‘Overdetermination’)

To detonate a certain bomb, either a or b must push their buttons. Both push their buttons simultaneously and the bomb detonates (k).

$$\tau \models [dstit\, a]k$$
$$\tau \models [dstit\, b]k$$

$$\tau \models \; [a]k \wedge [b]k \wedge \neg\Box k$$

One can define

$$[dstit!\, x]\varphi \;\; =_{\mathrm{def}} \;\; [x]\varphi \wedge \neg[Ag\smallsetminus\{x\}]\varphi$$

Collective/joint action (‘Underdetermination’)

For a different bomb both a and b must push their buttons simultaneously.

Generalises naturally to groups (sets) of agents:

$$[dstit\, G]\varphi =_{\text{def}} [G]\varphi \wedge \neg\Box\varphi$$

But (‘superadditivity’):

$$\models\ [dstit\, G]\varphi \to [dstit\, G']\varphi \qquad (G \subseteq G')$$

$$\models\ [dstit\, G]\varphi \to [dstit\, Ag]\varphi$$

Ag is a finite set of (unique names for) agents.

‘Strictly *stit*’: Contributors and bystanders

$\tau \models \Delta_G^{\mathsf{min}} \varphi$ iff G is a *minimal* non-empty set s.t. $\tau \models [G]\varphi$

iff G is a *minimal* set s.t. $\tau \models [dstit\, G]\varphi$

Δ_G^{min} corresponds to Belnap and Perloff’s ‘*strictly stit*’.

$$\models \ \Delta_G^{\mathsf{min}} \varphi \to [dstit\, G]\varphi$$

$$\models \ \Delta_{\{x\}}^{\mathsf{min}} \varphi \leftrightarrow [dstit\, x]\varphi$$

‘Strictly *stit*’: Contributors and bystanders

$\tau \models \Delta^{\mathsf{min}}_{G}\varphi$ iff G is a *minimal* non-empty set s.t. $\tau \models [G]\varphi$

iff G is a *minimal* set s.t. $\tau \models [\mathit{dstit}\, G]\varphi$

$\Delta^{\mathsf{min}}_{G}$ corresponds to Belnap and Perloff's ‘*strictly stit*’.

$$\models \ \Delta^{\mathsf{min}}_{G}\varphi \to [\mathit{dstit}\, G]\varphi$$

$$\models \ \Delta^{\mathsf{min}}_{\{x\}}\varphi \leftrightarrow [\mathit{dstit}\, x]\varphi$$

$$\tau \models \Delta^{\mathsf{max}}_{G^m}\varphi \quad \text{iff} \quad G^m = \bigcup_G \{\tau \models \Delta^{\mathsf{min}}_{G}\varphi\}$$

G^m are the *contributors* to φ at τ.
$Ag \setminus G^m$ are the ‘mere bystanders’ to φ at τ (Belnap and Perloff)

One can further distinguish ‘impotent bystanders’ to φ at τ and define $\Gamma_G\, \varphi$.

Example (‘Overdetermination’)

In order to detonate yet another bomb (k) any two of a b, c must push their buttons simultaneously. All three push their buttons.

$$\Delta^{\mathsf{min}}_{\{a,b\}} k \wedge \Delta^{\mathsf{min}}_{\{a,c\}} k \wedge \Delta^{\mathsf{min}}_{\{b,c\}} k$$

$$\Delta^{\mathsf{max}}_{\{a,b,c\}} k$$

Example ('Overdetermination')

In order to detonate yet another bomb (k) any two of a b, c must push their buttons simultaneously. All three push their buttons.

$$\Delta^{\text{min}}_{\{a,b\}} k \wedge \Delta^{\text{min}}_{\{a,c\}} k \wedge \Delta^{\text{min}}_{\{b,c\}} k$$
$$\Delta^{\text{max}}_{\{a,b,c\}} k$$

The Brexit referendum 2016

Example (‘Overdetermination’)

In order to detonate yet another bomb (k) any two of a b, c must push their buttons simultaneously. All three push their buttons.

$$\Delta^{\mathsf{min}}_{\{a,b\}} k \wedge \Delta^{\mathsf{min}}_{\{a,c\}} k \wedge \Delta^{\mathsf{min}}_{\{b,c\}} k$$
$$\Delta^{\mathsf{max}}_{\{a,b,c\}} k$$

The Brexit referendum 2016

- $[dstit\ Ag]\, leave$
- $\Gamma_G\, leave$ *those entitled to vote*
- $\Delta^{\mathsf{max}}_G\, leave$ *leavers + abstainers*
- $\Delta^{\mathsf{min}}_G\, leave$

Essential contributors

$$\tau \models \mathrm{Ness}_G\,\varphi \quad \text{iff } \tau \models [Ag]\varphi \wedge \neg[Ag \setminus G]\varphi$$

$$\tau \models \mathrm{Ness}_G^{\mathsf{min}}\,\varphi \quad \text{iff } G \text{ is a \textit{minimal} set s.t. } \tau \models \mathrm{Ness}_G\,\varphi$$

Cf. The '*NESS*' test in law: "*c* is a necessary element in a set of conditions sufficient for *e*".

Essential contributors

$$\begin{aligned} \tau \models \mathrm{Ness}_G\, \varphi \quad & \text{iff } \tau \models [Ag]\varphi \wedge \neg[Ag \smallsetminus G]\varphi \\ \tau \models \mathrm{Ness}^{\mathsf{min}}_G\, \varphi \quad & \text{iff } G \text{ is a } \textit{minimal} \text{ set s.t. } \tau \models \mathrm{Ness}_G\, \varphi \end{aligned}$$

Cf. The *'NESS'* test in law: "*c* is a necessary element in a set of conditions sufficient for *e*".

Proposition Suppose $\tau \models \Delta^{\mathsf{max}}_{G^m}\varphi$.

$$G^m \;=\; \bigcup_G \{\tau \models \underset{\substack{\uparrow \\ \text{minimal necessary}}}{\mathrm{Ness}^{\mathsf{min}}_G}\, \varphi \;\} \;=\; \bigcup_G \{\tau \models \underset{\substack{\uparrow \\ \text{minimal sufficient}}}{\Delta^{\mathsf{min}}_G}\, \varphi \;\}$$

Example: Bomb

Officer a: designated technicians $\{c, d\}$.
Officer b: designated technicians $\{c\}$.
To detonate this bomb, at least one officer and at least one of her designated technicians must press their buttons simultaneously.

pushers	$\Delta_G^{\mathsf{min}} k$	$\mathrm{Ness}_G^{\mathsf{min}} k$	$\Delta_G^{\mathsf{max}} k$
(a, b, c, d)	$\{a, c\}$, $\{a, d\}$, $\{b, c\}$	$\{a, b\}$, $\{a, c\}$, $\{c, d\}$	$\{a, b, c, d\}$
$(a, b, c, \cdot)$	$\{a, c\}$, $\{b, c\}$	$\{a, b\}$, $\{c\}$	$\{a, b, c\}$
$(a, b, \cdot, d)$	$\{a, d\}$	$\{a\}$, $\{d\}$	$\{a, d\}$
$(a, \cdot, c, d)$	$\{a, c\}$, $\{a, d\}$	$\{a\}$, $\{c, d\}$	$\{a, c, d\}$
$(a, \cdot, c, \cdot)$	$\{a, c\}$	$\{a\}$, $\{c\}$	$\{a, c\}$
$(a, \cdot, \cdot, d)$	$\{a, d\}$	$\{a\}$, $\{d\}$	$\{a, d\}$
$(\cdot, b, c, d)$	$\{b, c\}$	$\{b\}$, $\{c\}$	$\{b, c\}$
$(\cdot, b, c, \cdot)$	$\{b, c\}$	$\{b\}$, $\{c\}$	$\{b, c\}$

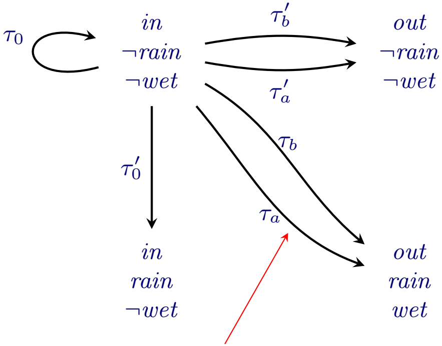
Indeterminism and ‘chancy causation’

Agents a and b can throw a certain precious vase out of a window. It might break or it might not. a throws the vase out the window and it breaks (k). Who broke the vase?

$$\tau \not\models [dstit\, a]k \qquad \tau \not\models [a]k$$

Agents a and b can also move the vase between inside and outside. If it is outside and it rains, the vase gets wet and is ruined. a moves the vase outside. It rains. Who ruined the vase?

Vase (two agents)



$$\tau_a \models [dstit\ a]\, out$$

$$\tau_a \not\models [dstit\ a]\, wet \qquad \tau_a \not\models [a]\, wet$$

$$\tau_a \not\models \Diamond [a] \neg wet$$

$$\tau_a \models \Diamond [a,b] \neg wet \qquad \tau_a \not\models \Diamond [a,b]\, wet$$

Two possible constructions

$[\overline{x}]\varphi$ — had x acted differently, it would have been φ
$\langle\overline{x}\rangle\varphi$ — had x acted differently, it might have been φ

$$\sim_{\overline{x}} =_{\mathrm{def}} \sim \setminus \sim_x$$

Two constructions:

$$\begin{array}{ll} (1) & \varphi \wedge [\overline{x}]\neg\varphi \\ (2) & \varphi \wedge \langle\overline{x}\rangle\neg\varphi \end{array}$$

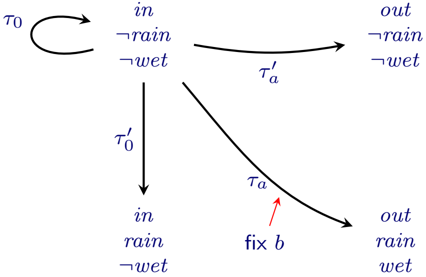
(1) is hopeless if there is more than one agent.
(2) is mentioned by Ingmar Pörn (1977) but is too weak.

$\varphi \wedge \Diamond[x]\neg\varphi$ does not work either

Chancy causation

But for a, *all things being equal*, it could have been different.

Suppose we *fix* the actions of all other agents (here b): b does not move the vase



In the *transformed* model:

$$\mathsf{fix}(Ag\setminus\{a\}), \tau_a \models \; wet \wedge \Diamond[a]\neg wet$$

Can we express this as a formula, without transforming the model?

Chancy causation

With actions of G_f fixed, G can guarantee φ

$$\langle G_f \rangle [G] \varphi \qquad \textit{no}$$

$$\langle G_f \rangle [G_f \cup G] \varphi$$

Chancy causation

With actions of G_f fixed, G can guarantee φ

$$\langle G_f\rangle[G]\varphi \qquad \textit{no}$$

$$\langle G_f\rangle[G_f \cup G]\varphi$$

With actions of $Ag\smallsetminus G$ fixed, G can guarantee φ

$$\langle Ag\smallsetminus G\rangle[(Ag\smallsetminus G) \cup G]\varphi$$

$$\langle Ag\smallsetminus G\rangle[Ag]\varphi$$

Chancy causation

With actions of G_f fixed, G can guarantee φ

$$\langle G_f \rangle [G] \varphi \qquad \textit{no}$$

$$\langle G_f \rangle [G_f \cup G] \varphi$$

With actions of $Ag \smallsetminus G$ fixed, G can guarantee φ

$$\langle Ag \smallsetminus G \rangle [(Ag \smallsetminus G) \cup G] \varphi$$

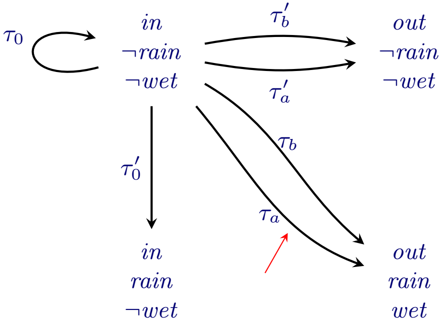
$$\langle Ag \smallsetminus G \rangle [Ag] \varphi$$

$$\mathrm{Could}_G\, \varphi \;\; =_{\mathrm{def}} \;\; \langle Ag \smallsetminus G \rangle [Ag] \varphi$$

A kind of responsibility for φ:

$$\varphi \wedge \mathrm{Could}_G \neg \varphi$$

(to be adjusted)



$\tau_a \sim_b \tau_0 \qquad \tau_a \sim_b \tau_0' \qquad \tau_a \sim_b \tau_a'$

$\tau_0 \models [a,b]\neg wet$

$\tau_a \models \langle b\rangle[a,b]\neg wet \qquad \{b\} = Ag \setminus \{a\} \qquad \tau_a \sim_b \tau_0$

$\tau_a \models wet \wedge \mathrm{Could}_{\{a\}}\neg wet$

$$\mathrm{Could}_G\,\varphi \;\;=_{\mathrm{def}}\;\; \langle Ag \smallsetminus G\rangle[Ag]\varphi$$

But ('superadditivity'):

$$\models\;\mathrm{Could}_G\,\varphi \to \mathrm{Could}_{G'}\,\varphi \qquad (G \subseteq G')$$

Define:

$\tau \models \mathrm{Could}^{\mathsf{min}}_G\,\varphi$ iff G is a *minimal* set s.t. $\tau \models \mathrm{Could}_G\,\varphi$

A kind of responsibility for φ:

$$\varphi \wedge \mathrm{Could}^{\mathsf{min}}_G \neg\varphi$$

Faulty bomb

$Ag = \{a, b, c\}$. If a and b together, or c on its own, or all three push their buttons then the bomb *might* detonate (k) or might not ($\neg k$); otherwise it will not detonate.

pushers					
(a, b, c)	k	$\mathrm{Could}^{\mathsf{min}}_{\{a,c\}} \neg k$	$\mathrm{Could}^{\mathsf{min}}_{\{b,c\}} \neg k$		
$(a, b, \cdot)$	k	$\mathrm{Could}^{\mathsf{min}}_{\{a\}} \neg k$	$\mathrm{Could}^{\mathsf{min}}_{\{b\}} \neg k$		
$(a, \cdot, c)$	k	$\mathrm{Could}^{\mathsf{min}}_{\{c\}} \neg k$			
$(\cdot, b, c)$	k	$\mathrm{Could}^{\mathsf{min}}_{\{c\}} \neg k$			
$(\cdot, \cdot, c)$	k	$\mathrm{Could}^{\mathsf{min}}_{\{c\}} \neg k$			
$(a, \cdot, \cdot)$	$\neg k$	$\Delta^{\mathsf{min}}_{\{b,c\}} \neg k$	$\mathrm{Ness}^{\mathsf{min}}_{\{b\}} \neg k$	$\mathrm{Ness}^{\mathsf{min}}_{\{c\}} \neg k$	
$(\cdot, b, \cdot)$	$\neg k$	$\Delta^{\mathsf{min}}_{\{a,c\}} \neg k$	$\mathrm{Ness}^{\mathsf{min}}_{\{a\}} \neg k$	$\mathrm{Ness}^{\mathsf{min}}_{\{c\}} \neg k$	
$(\cdot, \cdot, \cdot)$	$\neg k$	$\Delta^{\mathsf{min}}_{\{a,c\}} \neg k$	$\Delta^{\mathsf{min}}_{\{b,c\}} \neg k$	$\mathrm{Ness}^{\mathsf{min}}_{\{a,b\}} \neg k$	$\mathrm{Ness}^{\mathsf{min}}_{\{c\}} \neg k$

Cases where the bomb might detonate but does not are omitted.

Two kinds of responsibility or 'causes'

When $\tau \models [Ag]\varphi \wedge \neg\Box\varphi$:

$$\tau \models \Delta^{\mathsf{max}}_{G^m}\varphi$$

$$G^m \;=\; \bigcup_G\{\tau \models \mathrm{Ness}^{\mathsf{min}}_G\varphi\} \;=\; \bigcup_G\{\tau \models \Delta^{\mathsf{min}}_G\varphi\}$$

When $\tau \models \varphi \wedge \neg\Box\varphi$:

$$\tau \models \varphi \wedge \mathrm{Could}^{\mathsf{min}}_G\neg\varphi$$

How do they compare when $\tau \models [Ag]\varphi \wedge \neg\Box\varphi$?

Two kinds of responsibility or ‘causes’

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$$\tau \models \Delta^{\mathsf{max}}_{G^m}\varphi$$

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When $\tau \models \varphi \wedge \neg\Box\varphi$:

$$\tau \models \varphi \wedge \mathrm{Could}^{\mathsf{min}}_G\,\neg\varphi$$

How do they compare when $\tau \models [Ag]\varphi \wedge \neg\Box\varphi$?

$$\models\; [Ag]\varphi \to \big(\mathrm{Could}_G\,\neg\varphi \to \mathrm{Ness}_G\,\varphi\big)$$
$$\models\; [Ag]\varphi \to \big(\mathrm{Could}^{\mathsf{min}}_G\,\neg\varphi \to \mathrm{Ness}_G\,\varphi\big)$$
$$\models\; [Ag]\varphi \to \big(\mathrm{Could}^{\mathsf{min}}_{\{x\}}\,\neg\varphi \to \mathrm{Ness}^{\mathsf{min}}_{\{x\}}\,\varphi\big) \qquad (x \in Ag)$$

Chancy causation: Formulation 2

Formulation 1

$$\varphi \land \mathrm{Could}_G \neg\varphi \ = \ \varphi \land \langle Ag \setminus G \rangle [Ag] \neg\varphi$$

With actions of $Ag \setminus G$ fixed, Ag could have ensured $\neg\varphi$

Formulation 2

$$\varphi \land \Diamond [Ag] \neg\varphi \land \neg\langle G \rangle [Ag] \neg\varphi$$

Ag have a way of ensuring $\neg\varphi$ but with actions of G fixed, Ag have no way of ensuring $\neg\varphi$.

Chancy causation: Formulation 2

Formulation 1

$$\varphi \wedge \mathrm{Could}_G \neg\varphi \;=\; \varphi \wedge \langle Ag \smallsetminus G \rangle [Ag] \neg\varphi$$

With actions of $Ag \smallsetminus G$ fixed, Ag could have ensured $\neg\varphi$

Formulation 2

$$\varphi \wedge \Diamond [Ag] \neg\varphi \wedge \neg\langle G \rangle [Ag] \neg\varphi$$

Ag have a way of ensuring $\neg\varphi$ but with actions of G fixed, Ag have no way of ensuring $\neg\varphi$.

$$\varphi \wedge \Diamond [Ag] \neg\varphi \wedge \neg\langle G \rangle [Ag] \neg\varphi$$
$$\varphi \wedge \neg\Box \langle Ag \rangle \varphi \wedge [G] \langle Ag \rangle \varphi$$
$$\varphi \wedge [dstit\, G] \langle Ag \rangle \varphi$$

and for *minimal* such

$$\varphi \wedge \Delta_G^{\mathsf{min}} \langle Ag \rangle \varphi$$

‘Chancy *stits*’

$$[G]^{\epsilon}\varphi \;=_{\mathrm{def}}\; \varphi \wedge [G]\langle Ag\rangle\varphi$$

$$[dstit^{\epsilon}\, G]\varphi \;=_{\mathrm{def}}\; \varphi \wedge [dstit\, G]\langle Ag\rangle\varphi \qquad ([G]^{\epsilon}\varphi \wedge \neg\Box\langle Ag\rangle\varphi)$$

$$\Delta_G^{\epsilon\text{-}\mathsf{min}}\varphi \;=_{\mathrm{def}}\; \varphi \wedge \Delta_G^{\mathsf{min}}\langle Ag\rangle\varphi$$

‘Chancy *stits*’

$$[G]^{\epsilon}\varphi \;=_{\mathrm{def}}\; \varphi \wedge [G]\langle Ag\rangle\varphi$$

$$[dstit^{\epsilon}\, G]\varphi \;=_{\mathrm{def}}\; \varphi \wedge [dstit\, G]\langle Ag\rangle\varphi \qquad ([G]^{\epsilon}\varphi \wedge \neg\Box\langle Ag\rangle\varphi)$$

$$\Delta_G^{\epsilon\text{-}\mathsf{min}}\varphi \;=_{\mathrm{def}}\; \varphi \wedge \Delta_G^{\mathsf{min}}\langle Ag\rangle\varphi$$

We know $\bigcup_G\{\tau \models \Delta_G^{\mathsf{min}}\langle Ag\rangle\varphi\} \;=\; \bigcup_G\{\tau \models \mathrm{Ness}_G^{\mathsf{min}}\langle Ag\rangle\varphi\}$

Lemma $\models \varphi \to (\,\mathrm{Could}_G\,\neg\varphi \;\leftrightarrow\; \mathrm{Ness}_G\langle Ag\rangle\varphi\,)$

Proposition If $\tau \models \varphi$ then

$$\bigcup_G\{\tau \models \Delta_G^{\epsilon\text{-}\mathsf{min}}\varphi\} \;=\; \bigcup_G\{\tau \models \mathrm{Could}_G^{\mathsf{min}}\,\neg\varphi\}$$

Example: Firing squad

Suppose (pretend) $\{a, b, c, d, e, f, g, h\}$ all fire *simultaneously*.

If every individual shot is guaranteed to be fatal:

$$\Delta^{\mathsf{min}}_{\{a\}} k \wedge \Delta^{\mathsf{min}}_{\{b\}} k \wedge \cdots \ \wedge \Delta^{\mathsf{min}}_{\{h\}} k$$

$$\Delta^{\mathsf{max}}_{\{a,b,c,d,e,f,g,h\}} k$$

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If every combination of three shots (say) is needed and guaranteed to be fatal:

$$\Delta^{\text{min}}_{\{a,b,c\}} k \wedge \Delta^{\text{min}}_{\{a,b,d\}} k \wedge \cdots \ \wedge \Delta^{\text{min}}_{\{f,g,h\}} k$$
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If individual shots are not necessarily fatal but on this occasion it happens that all of them were:

$$k \wedge \mathrm{Could}^{\mathsf{min}}_{\{a,b,c,d,e,f,g,h\}} \neg k$$

$$\Delta^{\epsilon\text{-}\mathsf{min}}_{\{a\}} k \wedge \Delta^{\epsilon\text{-}\mathsf{min}}_{\{b\}} k \wedge \cdots \ \wedge \Delta^{\epsilon\text{-}\mathsf{min}}_{\{h\}} k$$

Causal dependencies and *stit*

j stands on top of a high building. j may jump or not. If j jumps, a sniper s may shoot j, or may not. s cannot shoot j if j does not jump.

(j, s)	k	$\Delta^{\mathsf{min}}_{\{j\}}k,\ \Delta^{\mathsf{min}}_{\{s\}}k$
$(j, \cdot)$	k	$\Delta^{\mathsf{min}}_{\{j\}}k$
$(\cdot, \cdot)$	$\neg k$	$\Delta^{\mathsf{min}}_{\{j\}}\neg k$

Deterministic

(j, s)	k	$\Delta^{\epsilon\text{-}\mathsf{min}}_{\{j\}}k,\ \Delta^{\epsilon\text{-}\mathsf{min}}_{\{s\}}k$
$(j, s)'$	$\neg k$	
$(j, \cdot)$	k	$\Delta^{\epsilon\text{-}\mathsf{min}}_{\{j\}}k$
$(j, \cdot)'$	$\neg k$	
$(\cdot, \cdot)$	$\neg k$	$\Delta^{\mathsf{min}}_{\{j\}}\neg k$

Indeterministic

Other combinations (jumping guaranteed fatal, shooting not), (shooting guaranteed fatal, jumping not) are also possible.

Example: jumper and screamer

s is not a sniper but a mere observer. If j jumps, s may scream in alarm, or may not. s does not scream if j does not jump.

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Same model!

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$(j,\cdot)$	k	$\Delta^{\text{min}}_{\{j\}}k$
$(\cdot,\cdot)$	$\neg k$	$\Delta^{\text{min}}_{\{j\}}\neg k$

(j,s)	k	$\Delta^{\epsilon\text{-min}}_{\{j\}}k,\ \Delta^{\epsilon\text{-min}}_{\{s\}}k$
$(j,s)'$	$\neg k$	
$(j,\cdot)$	k	$\Delta^{\epsilon\text{-min}}_{\{j\}}k$
$(j,\cdot)'$	$\neg k$	
$(\cdot,\cdot)$	$\neg k$	$\Delta^{\text{min}}_{\{j\}}\neg k$

In particular:

$(j,s) \models \Delta^{\text{min}}_{\{s\}}k$

$(j,s) \models \Delta^{\epsilon\text{-min}}_{\{s\}}k$

How can this be?

Example: jumper and screamer (modified)

j reacts to s not the other way round. j may jump or not, but a scream from s always triggers a jump from j.

Same model again.

(j, s)	k	$\Delta^{\mathsf{min}}_{\{j\}} k,\ \Delta^{\mathsf{min}}_{\{s\}} k$
$(j, \cdot)$	k	$\Delta^{\mathsf{min}}_{\{j\}} k$
$(\cdot, \cdot)$	$\neg k$	$\Delta^{\mathsf{min}}_{\{j\}} \neg k$

(j, s)	k	$\Delta^{\epsilon\text{-}\mathsf{min}}_{\{j\}} k,\ \Delta^{\epsilon\text{-}\mathsf{min}}_{\{s\}} k$
$(j, s)'$	$\neg k$	
$(j, \cdot)$	k	$\Delta^{\epsilon\text{-}\mathsf{min}}_{\{j\}}$
$(j, \cdot)'$	$\neg k$	
$(\cdot, \cdot)$	$\neg k$	$\Delta^{\mathsf{min}}_{\{j\}} \neg k$

In particular:

$(j, s) \models \Delta^{\mathsf{min}}_{\{s\}} k$

$(j, s) \models \Delta^{\epsilon\text{-}\mathsf{min}}_{\{s\}} k$

Seems reasonable now?

Not really so surprising . . .

In the example

$$\models\ \neg\Diamond(s{:}shouts \wedge \neg j{:}jumps)$$

And so:

$$\models\ \Box(s{:}shouts \rightarrow j{:}jumps)$$
$$\models\ \Box(\neg j{:}jumps \rightarrow \neg s{:}shouts)$$

But of course these do not express *causal* connections'

And then e.g.

$$\models\ [s]\varphi \rightarrow [j]\varphi$$

But that is not a *causal* connection either

Causal dependencies (perhaps)

Fix $G_f \subseteq Ag \setminus G$ whose actions are *not causally dependent* on G.

$$\langle G_f \rangle [G_f \cup G] \varphi$$

If s depends on j:

$(j, s) \models k \wedge \langle \emptyset \rangle [j] \neg k$ $\quad$ j responsible

$(j, s) \not\models k \wedge \langle j \rangle [j, s] \neg k$

If j depends on s:

$(j, s) \not\models k \wedge \langle s \rangle [j, s] \neg k$

$(j, s) \not\models k \wedge \langle \emptyset \rangle [s] \neg k$

$(j, s) \models k \wedge \langle \emptyset \rangle [j, s] \neg k$ $\quad$ j, s jointly responsible

Perhaps this might work — but it does not fit in the *stit* semantics

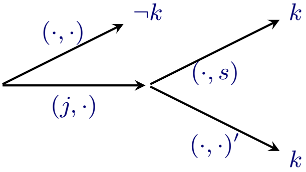
The conclusion?

$\tau \models \Delta_G^{\epsilon\text{-min}} \varphi$ and $\tau \models \mathrm{Could}_G^{\mathsf{min}} \varphi$ express what we want if at τ the actions of agents are *causally independent*.

stit-independence — all actions of distinct agents are causally independent at all τ

Temporal refinement

jumper-screamer (jumps fatal)



branches

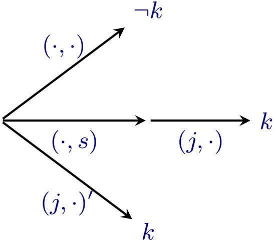
$(\cdot,\cdot)$	$\neg k$
$(j,\cdot)\,;(\cdot,s)$	k
$(\cdot,s)$	k
$(j,\cdot)\,;(\cdot,\cdot)'$	k
$(\cdot,\cdot)'$	k

stit-independence at each branch point

$$\begin{aligned} (j,\cdot)\,;\,(\cdot,s) &\models\ [j]k \wedge \neg\Box k \wedge \neg[s]k \\ (\cdot,s) &\models\ [s]k \wedge \Box k \end{aligned}$$

Temporal refinement

screamer-jumper (jumps fatal)



branches

$(\cdot,\cdot)$	$\neg k$
$(\cdot,s)\ ;(j,\cdot)$	k
$(j,\cdot)$	k
$(j,\cdot)'$	k

$$\begin{aligned} (j,\cdot)' &\models\ [j]k \wedge \neg\Box k \\ (\cdot,s)\,;\,(j,\cdot) &\models\ [s]k \wedge \neg\Box k \wedge \neg[j]k \\ (j,\cdot) &\models\ [j]k \wedge \Box k \end{aligned}$$

Details: many, many variations!!

Conclusion

- $\Delta_G^{\mathsf{min}}\varphi$ and $\mathrm{Ness}_G^{\mathsf{min}}\,\varphi$
  $\Delta_G^{\mathsf{min}}\langle Ag\rangle\varphi$ and $\mathrm{Could}_G^{\mathsf{min}}\,\neg\varphi$
- Causal dependencies are not modelled in *stit*
- Temporal extensions are essential, for many reasons.
  There are many possible variations to be explored.
- Temporally refined models are not a panacea
  Sometimes we treat sequential actions as if they were simultaneous.

  And then *stit*-independence is not wanted (in my opinion).
- *stit* can be extended with act types
  That is essential anyway

  (Is it then still *stit* ?)